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# TRANSPORT PHENOMENA

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## Preface

This book is intended to present the subjects of molecular transport (heat conduction, convection, and diffusion). In this treatment, the molecular and continuum approaches are compared, although it should be emphasized that the book is not intended for complete mastery of the subjects.

Because of the current emphasis on understanding the use of empiricism, we feel the book is of a different kind. Obviously the subject is treated across traditional departmental boundaries. "The subject of transport phenomena is the common ground of mechanics, and electromagnetism." Knowledge of the subject of transport has certainly become an essential part of engineering analysis. In addition, the subject has great interest to some who are working in meteorology, and biology.

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